
A Phase Wraps Reduction Method Based on Fringe Pattern

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Abstract:

In fringe projection profilometry, wrapped phase is useless, the phase wraps are eliminated completely or the number of phase wraps is greatly reduced, which is helpful for subsequent phase unwrapping. Thus, an efficient method is proposed to absolutely remove phase wraps or tremendously decrease the number of phase wraps. Firstly, the sub-pixel peak location of spectrum is determined by two 1D iterative local Fourier transform; then the frequency spectrum moved to the origin in the spatial domain. After this, the phase wraps can be rapidly and accurately reduced. Finally, the proposed method is proved by simulated and actual strike images, and compared with the spectrum is shifted in the frequency domain and the spectrum is shifted in the spatial domain after zero-padding, proved the efficiency of the proposed method.

Keywords: Fringe projection analysis, Phase wraps, Phase extracting, Spectrum peak estimation, Fourier transform.

I. INTRODUCTION

Three dimensional(3D) profile measurement of objects is widely used in aerospace, automobile, agricultural machinery manufacturing, human body measurement, medical treatment, 3D printing and so on[1,2]. In recent years, fringe projection profilometry based on phase measurement is widely used in 3D topography measurement of the object [3,4]. Phase calculation includes phase extraction and phase unwrapping, the common phase extraction methods are as follows: phase shifting method [5], Fourier transformation [6] and Wavelet transform method [7]. No matter which extraction method is used, the phase information is mathematically expressed using the arc tangent function, so the extracted phase is wrapped in the section $(-\pi, \pi)$, phase information is not continuous and known as the wrapped phase. Wrapping phase is useless, hence, phase unwrapping method is used to dislodge phase wraps and to obtain continuous phase. In recent years, a wide variety of phase unwrapping algorithms have been proposed. These algorithms include branch cutting method, quality guidance method, least square method, minimum norm method and so on spatial methods, and multi-frequency

heterodyne time algorithm. The spatial phase unwrapping's method is a process of integral accumulation, as a pixel is wrong, it will spread to all the pixels behind, and affect the quality of phase unwrapping. For example, the noise, the occlusion, the surface abrupt change and discontinuity of the object in the fringe image will influence the quality of phase unwrapping. The method of temporal phase unwrapping require that the gratings of different frequencies are projected onto the surface of the object to be measured, the deformed grating images different frequencies are spread in time domain, the phase of each pixel in the images is unwrapped along the time axis independently, this can avoid the error propagation[8]. However, the time phase unwrapping method needs a lot of fringe images, which will take a lot of time and is not suitable for dynamic and real-time 3D measurement. Therefore, although a large number of phase unwrapping methods are proposed, no one method can solve all the problems. Phase unwrapping has been a difficult problem for scholars in recent years.

For avoiding the phase unwrapping process or simplify the phase unwrapping complexity and accelerate the phase unwrapping process, the researchers at home and abroad have proposed many algorithms to absolutely remove or decrease the number of phase wraps in the wrapped phase image, which will certainly help for the phase unwrapping step. In 2011, Gdeisat [9] proposed a phase unwrapping algorithm to absolutely remove or tremendously decrease the number of phase wraps, if the frequency spectrum shifting is an integer, all phase wraps are eliminated, and the phase unwrapping step can be omitted. However, the method uses a discrete Fourier transform, which will limit the spectrum shift to integer values in all case. In 2016, Gdeisat et al. [10] proposed a spectrum non integer shift phase wraps reduction algorithm. The wrapped phase image is padded by zeros and the resolution of Fourier transform is increased, and the non-integer shift of the spectrum is realized. The zero padding method will occupy a large amount of computing time and large storage capacity. In addition, the frequency domain algorithm needs Fourier transform, spectrum determination, spectrum shift, inverse Fourier Transform and so on steps, these steps will increase the computational complexity and processing time. In 2016, Herráez et al. [11] proposed a spatial wrap reduction method in view of the mode estimation. Firstly, the mode of the distributions of wrapped first differences is estimated from each axis, and an oblique plane is calculated, then the slope is subtracted from the wrapped phase, and rewrapped the result to obtain a new phase image with removing the most wraps. This method does not require the positive and inverse Fourier transform, improves the calculation speed, but the accuracy is lower than the frequency domain.

Aiming at the shortcomings of the above methods, this paper proposes an effective spatial method to eliminate or decrease phase wraps. Firstly, the location of the spectrum peak is determined to sub-pixel accuracy by iterative 1D local Fourier transform in each axis. Secondly, the non-integer spectrum shift is achieved in space domain to absolutely remove or tremendously decrease the number of the phase wraps. The proposed method achieves the accuracy of spectrum shift in frequency domain, while the speed is significantly improved than it. The phase wraps reduction algorithm can be combined with other phase unwrapping methods

to obtain fast and high-precision 3D profile measurement. Finally, the effectiveness of phase wraps reduction algorithm and the potential benefits of phase unwrapping are verified by simulated and actual fringe images.

II. PRINCIPLE OF PHASE WRAPPING REDUCTION METHOD

The four step phase shifting stripe images are captured by the camera as shown in formula (1).

$$I_d = \cos(2\pi f_x x + 2\pi f_y y + \beta\varphi(x, y) + (d-1)\pi/2) \quad d=1,2,3,4 \quad (1)$$

Where f_x and f_y are the space carrier frequency along the X and Y axis respectively, β is modulate index. The wrapped phase is extracted by the famous four step phase shifting method, the result is shown as (2).

$$\varphi_w(x, y) = \arg \tan \frac{I_4 - I_2}{I_3 - I_1} = W(2\pi f_x x + 2\pi f_y y + \beta\varphi(x, y)) \quad (2)$$

The wrapped phase is wrapped in $(-\pi, \pi)$.

In fringe pattern, the carrier frequency is transformed into a linear and monotonic component in the phase information, resulting in an increase in the number of wraps. Therefore, removing or decreasing the carrier frequency will eliminate the phase wraps or reduce the number of the phase wraps, and avoid the phase unwrapping process, or simplify and accelerate the phase unwrapping process.

2.1 Wrap Reduction algorithm in frequency domain

AS described in Ref.9. The wrapped phase is transformed into complex matrix $\varphi_{wc}(x, y)$, the result is shown as equation (3).

$$\varphi_{wc}(x, y) = e^{j\varphi_w(x, y)} \quad (3)$$

The Fourier transform of the $\varphi_{wc}(x, y)$ is shown as equation (4).

$$\Phi_{wc}(u, v) = F(\varphi_{wc}(x, y)) \quad (4)$$

The peak position of $\Phi_{wc}(u, v)$ is calculated, that is, the distance of the spectrum shifting to the origin are shown as equation (5).

$$u_0, v_0 = \max(\text{abs}(\Phi_{wc}(u, v))) \quad (5)$$

Moving the spectrum to the origin as shown $\Phi_{wc}(u - u_0, v - v_0)$. Then the inverse Fourier transform is carried out for the moved spectrum, shown as equation (6).

$$\psi(x, y) = F^{-1}(\Phi_{wc}(u - u_0, v - v_0)) \quad (6)$$

Then, the Arctangent () is used to extract phase, the result is shown as equation (7).

$$\phi_{wcs}(x, y) = \arg \tan \left(\frac{R(\psi(x, y))}{I(\psi(x, y))} \right) = W(2\pi(f_x - u_0)x + 2\pi(f_y - v_0)y + \beta\varphi(x, y)) \quad (7)$$

From the formula (7) can be seen that in the frequency domain, the frequency spectrum moves towards the origin, which is equivalent to reducing the space carrier frequency of the fringe image. Therefore, the number of phase wraps will be reduced in the new phase image [9].

The above spectral shift distances u_0 and v_0 are only integers, but in most of the actual application, the spectral shift distances are not an integer. For solving the problem of non-integer spectrum shift, literature [10] improved the above method, non-integer shift in frequency domain is realized by zero padding method. Firstly, the wrapped phase map is padded by zeros for both the horizontal direction and the vertical direction [10], shown as equation (8).

$$\varphi_{wz}(x, y) = \begin{cases} \varphi_w(x, y) & 0 \leq x \leq M-1, 0 \leq y \leq N-1 \\ 0 & M \leq x \leq kM-1, N \leq y \leq kN-1 \end{cases} \quad (8)$$

If setting $k=10$, the calculation accuracy of the peak is increased by 10 times [12]. However, when the k value is too large, the fringe image will become too large after the image is padded by the zero, then the Fourier transform, search and shift spectrum and inverse Fourier transform and so on, the computing time becomes quite long and occupies a larger storage space.

2.2 Shift the spectrum in spatial domain

According to the above calculation u_0 and v_0 , the spectrum shifting can be realized in the space domain according to the frequency conversion characteristic of the 2D Fourier transform [13], and the spectral transform characteristic of 2D Fourier is shown in formula (9).

$$F(u - u_0, v - v_0) \Leftrightarrow f(x, y)e^{j2\pi(u_0x/M_x + v_0y/N_y)} \quad (9)$$

Where M_x and N_y are the row and column number of the fringe image. According to the formula (9), the spectrum shifting can be realized by multiplication operations in the space domain, such as the formula (10).

$$\psi(x, y) = \varphi_{wc}(x, y)e^{-j(2\pi u_0x/M_x + 2\pi v_0y/N_y)} = e^{j(\varphi_w(x, y) - 2\pi u_0x/M_x - 2\pi v_0y/N_y)} \quad (10)$$

A new phase map of wraps reduction is obtained according to (7) formula. The space domain wraps reduction method does not need the positive and inverse Fourier transform, search and move spectrum, simplify the frequency domain process, save computing time.

2.3 Proposed wraps reduction algorithm

Based on the shortcomings of the above methods, a sub pixel precision 1D iterative local Fourier transform algorithm is proposed to determine the u_0 and v_0 , and then the spectrum shifting is implemented in the spatial domain to achieve fast and efficient elimination or reduce the phase wraps effect. For determining the frequency of fringe patterns, many methods are proposed. Commonly used mathematical fitting methods, such as: parabola fitting spectrum peak method [14] and the centroid algorithm of the grid around spectral peak [15], the estimation results of these methods depend on the grid size, the number of fitting parameters and so on fitting conditions, and the accurate frequency estimation depends on the creation of the fitting model. Another common method is zero padding method [10,12], when the original

image is padded with zero, the more the number of elements padded, the higher the accuracy of frequency estimation, but the large image increases the computing time and storage space. In 2016, S.Lee et al. [16] proposed an iterative 2D local Fourier transform algorithm to estimate the frequency. The 2D iterative algorithm has high accuracy and speed, but the computation complexity can reach $O(\max(M,M)^3)$ if it is iterated in the local area of $M \times M$.

For reducing the high computational burden caused by the 2D iterative local Fourier transform frequency estimation method, 2D iterative local Fourier transform frequency estimation method is improved, two 1D iterative local Fourier transforms are used to estimate the frequency from each axis to replace the 2D iterative local Fourier transform frequency estimation. The main steps are as follows:

(1) Take the middle row of the $\varphi_{wc}(x, y)$, make 1D fast Fourier transform to determine an initial spectrum peak position;

(2) Interval length 5 local frequency coordinates around the spectrum peak position are formed with the coordinates sampled at 1/2 of the initial fast Fourier transform frequency interval;

(3) Discrete Fourier transform is used to calculate the frequency spectrum value and a new spectrum peak position is searched among the generated 10 spectrum values;

(4) Repeat step (2) (3), and obtain a sub-pixel estimation for the spectral peak position u_0 .

Repeat the same process above to get the v_0 .

Each iteration is repeated, the accuracy of frequency peak position is improved by 2 times, and five iterations improve the accuracy of the spectral peak position by 2^5 times compared to the initial resolution. The more is iterations, the higher is the accuracy.

III. COMPUTER SIMULATION

The spectrum shifting in the frequency domain, the spectrum shifting in the spatial domain and the proposed method are verified by the analog images. The simulation runs on the same machine, with 2.30GHz Intel Pentium (R) dual core CPU T4500 and 8GB memory. A 3D object is created by the peaks () in Matlab, the size of 512×512 pixels, shown as Fig 1(a), the corresponding fringe pattern is obtained by formula (1), including $f_y=0, f_x=1/16, \beta=1$, one of the fringe images is shown as Fig 1(b), the wrapped phase is extracted by formula (2), shown as Fig 1(c). Fig 1(c) contains many wraps, and should be removed by phase unwrapping algorithm. Phase wraps reduction algorithm is used to eliminate or reduce phase wraps before phase unwrapping.

The principle of reducing wraps by spectrum shifting in frequency domain, the moving distance is determined by the position of the fundamental spectrum peak, $u_0=512/16=32, v_0=0$ are integer, the spectrum moves 32 pixels to the origin, as shown in Fig 1(d), according to formula(7), the new wrapped phase map is shown as Fig 1(e), the multiplication in the spatial domain can also be used to realize the spectrum shifting, the result is shown in Fig 1(f). From Fig 1(e) and (f), the phase maps obtained by the two methods are almost the same, but the

execution time of the two methods is different, as shown in TABLE I. It can be seen that in this case, the wraps reduction algorithm has completely eliminate the wraps, and avoid the phase unwrapping process.

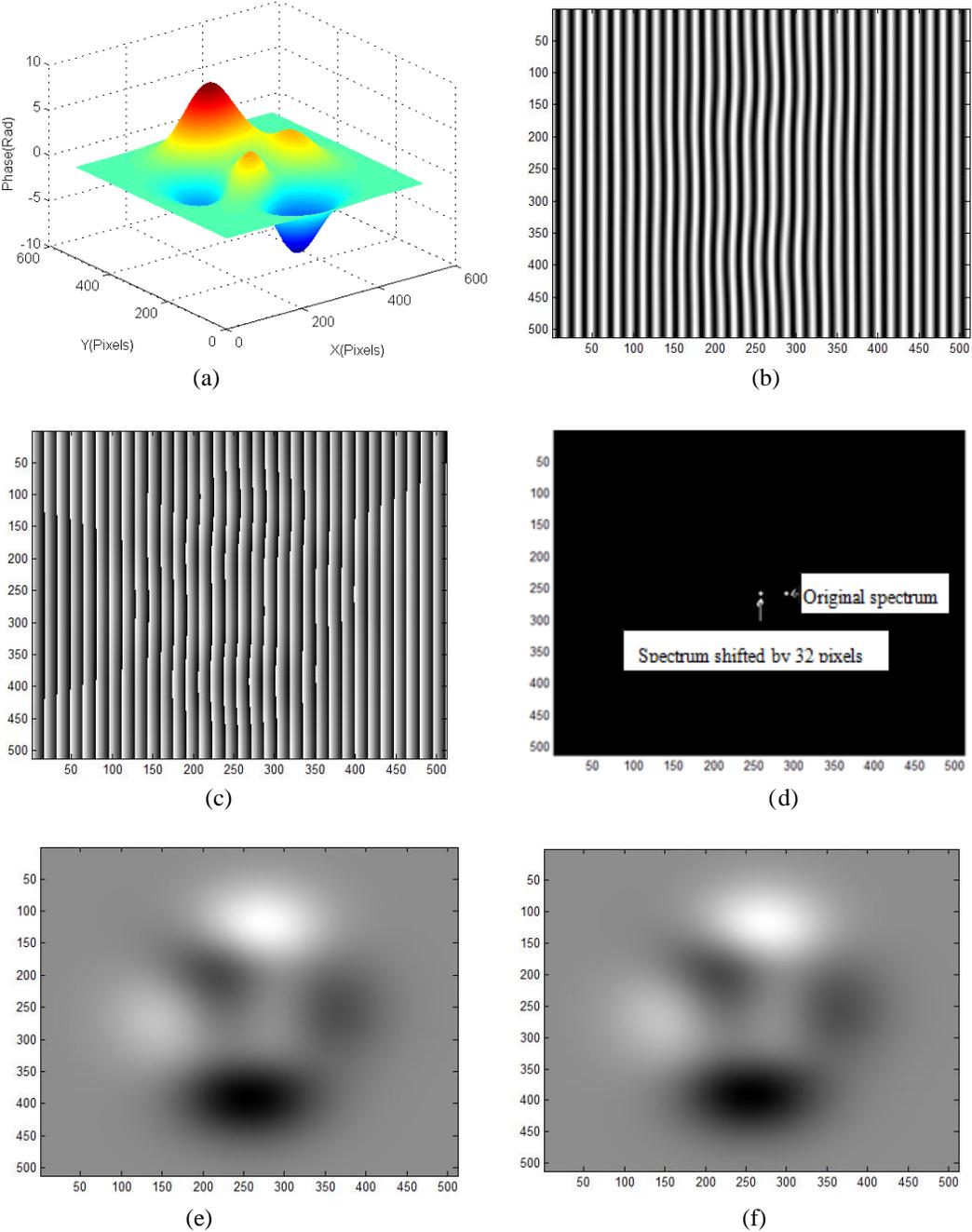


Fig 1: The spectrum moving distance is an integer in the frequency domain (a) The 3D simulation object (b) A simulated object fringe image (c) The wrapped phase map of the simulated object (d) The frequency spectrum of the wrapped phase and the new frequency

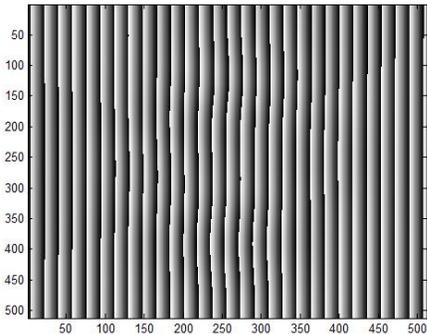
spectrum by the peak is shifted to the origin (e) The new phase map by frequency domain (f) The new phase map by spatial domain

TABLE I. Comparisons of executing time of the two methods

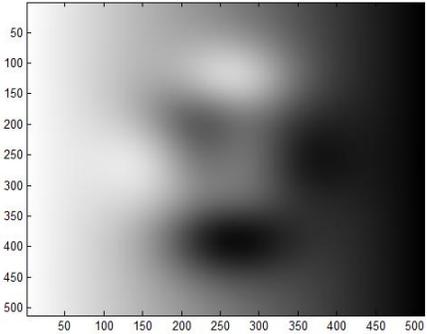
Method Name	Total executing time
The shifted spectrum in frequency domain	0.9812s
The shifted spectrum in spatial domain	0.5178s

In the example above, if the spectrum shifting is just an integer, the wraps are completely eliminated, and the result is ideal. The space algorithm avoids the positive and inverse Fourier transformation and the selection and shifting of the spectrum, therefore, execution time reduces nearly half of the frequency domain algorithm.

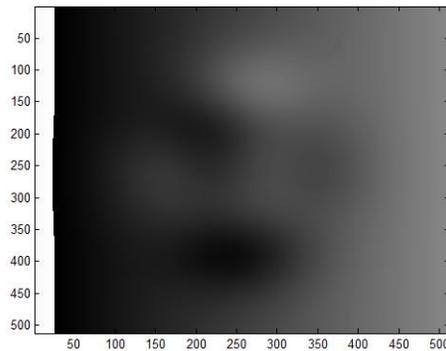
Secondly, spectrum shifting is non-integer, the size of 512×512 pixels simulation object is repeat, but with the space carrier frequency $f_x=1/18, f_y=0, \beta=1$, the wrapped phase image is shown as Fig 2(a), the frequency spectrum will be moved to the origin by a distance $u_0=512/18=28.4444$, but according to document [21], the spectrum shifting can only an integer, the frequency spectrum is moved here by the values $u_0=28$ and $v_0=0$, the new wrapped phase map is shown as Fig 2(b), if the frequency spectrum is moved here by the values $u_0=29$ and $v_0=0$, the new wrapped phase image is shown as Fig 2(c). In Fig 2 (b) (c), the wraps still exists, because the spectrum shifting distance is not integer in this case.



(a)



(b)



(c)

Fig 2: The spectrum moving distance is a non-integer in the frequency domain (a) The wrapped phase image of the simulation object (b) The new phase map by shifting the spectrum with 28 pixels (c) Obtained the new phase map by the spectrum is shifted 29 pixels

In order to realize non integer shift, a zero padding method is proposed in document [10][16]. Setting $k=10$, the accuracy is improved by 10 times[10][16], firstly, the wrapped phase map is padded by zero, the image size is 10 times of the original image, namely 5120×5120 pixels, shown as Fig 3(a). The padded wrapped phase image is transformed into the complex matrix by formula (3) (4) (5), and the position of the spectrum peak is obtained at $[2845, 2561]$, So the spectrum shift distance are $u_0=2845-2561=284$ and $v_0=0$, as shown in Fig 3(b), a new phase image is obtained by moving the frequency spectrum in the space domain, shown as Fig 3(c), the phase image was cut into 512×512 pixels, as shown in Fig 3(d), the accuracy of shift distance can reach 28.4 of the frequency domain.

Finally, the proposed two 1D local iterative Fourier transform method is used to determine u_0 and v_0 , and then spectrum shifting is implemented in the spatial domain. Take the 256th row in Fig 2(a), the initial position of spectral peak is $(285, 257)$ by Fast Fourier transform, shift distance are $u_0=285-257=28$ and $v_0=0$, the 1/2 coordinate of the last frequency interval is taken in each iteration, 5 iterations are made in the local range of 5 pixels, estimated $u_0=285.4432-257=28.4432$, according to u_0 , the spectrum shifting is completed by multiplication operation in the space domain, as shown in Fig 3(e). Fig 3(d) and (e) look similar, but the execution time varies greatly, and the execution time of the two methods is shown in TABLE II, execution time includes zero padding and cropping time. In the space domain, the multiplication operation realizes the spectrum shifting, because of avoiding the positive and inverse transformation of Fourier, the selection and shifting of the frequency spectrum, the computing speed has been improved. However, since the zero-padding map becomes very large, the speed is still much lower than the proposed iterative 1D Fourier transform to estimate the spectral peaks.

The phase wraps can be eliminated by zero padding and the proposed method in this event, subsequent phase unwrapping is not needed. Because the simulated image does not consider the light and noise in the actual scene, it is more ideal to reduce the phase wraps effect by using the wraps reduction algorithm.

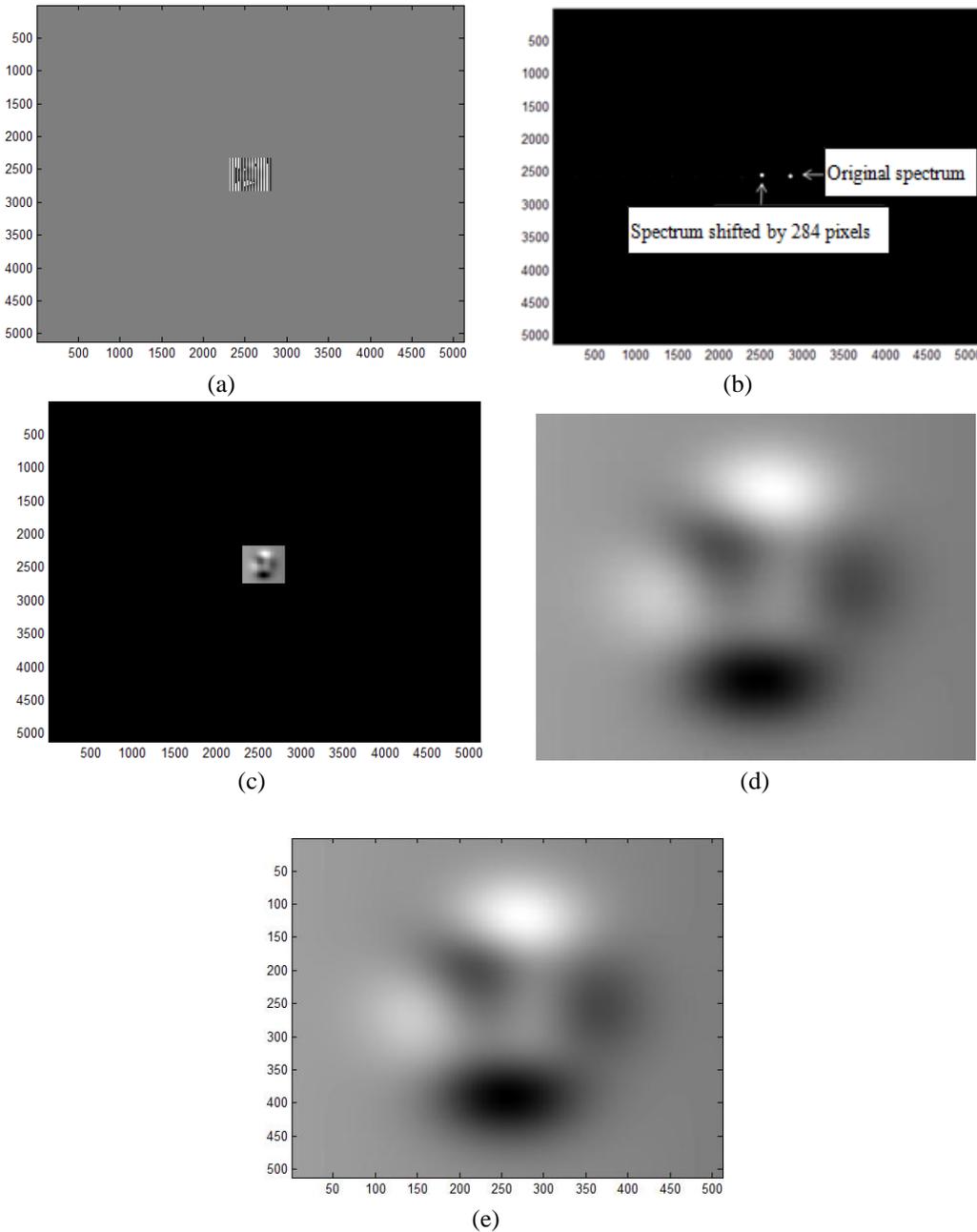


Fig 3: The spectrum moving distance is a non-integer (a) The zero padded wrapped phase image (b) The original frequency spectrum map of the wrapped phase and the new frequency spectrum map by the peak is shifted to the origin (c) The new phase map by shifting the spectrum with 284 pixels (d) The phase map is cut to the size of 512×512 pixels (e) Obtained the phase map by the proposed method

TABLE II. Comparisons of executing time of the two algorithms

Algorithm Name	Total executing time
The zero-padding method	5.606s
The proposed method	2.859s

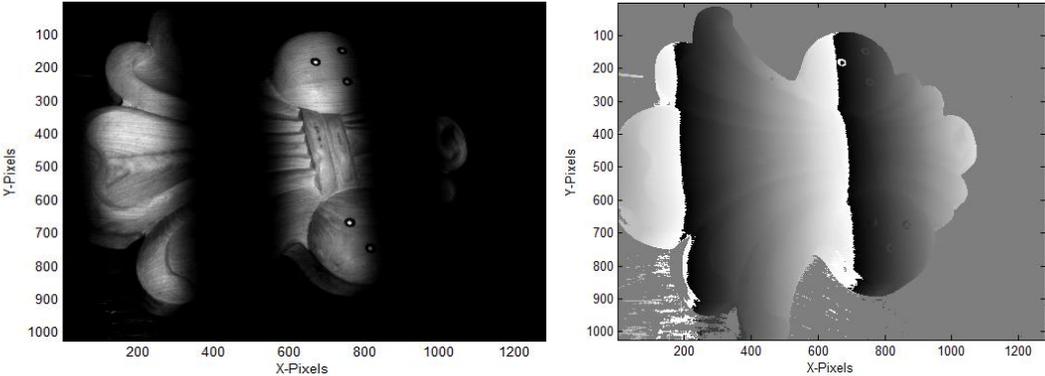
IV. EXPERIMENT

Here, the measurements of two real objects are run to verify the potential benefits of the proposed wraps reduction algorithm for phase unwrapping. The first one is a relief model, and the other is a nut. The structured light measurement system is shown in Fig 4, 1 Sharp A8X digital projectors, 2 Basler A102F CCD cameras with a resolution of 1280 x 1024 and a nominal focal length of 16mm.

The Relief fringe image is shown as Fig 5(a), extracted the wrapped phase map is shown as Fig 5(b), the spectrum is shifted in the frequency domain for the wrapped phase image, the result is shown as Fig 5(c). Fig 5(c) shows that the phase wraps are completely eliminated because the spectrum shift distance is just integer $u_0=4$. According to the shifting distance 4, the spectrum is shifted by multiplication in the spatial domain, and the result is the same as that of Fig 5(c). However, the implementation time of the two methods is different, the spectrum shifting method in frequency domain takes 2.0167s time, and in the space domain the spectrum shifting method takes 0.9596s time.



Fig. 4. The measurement system based on structured light



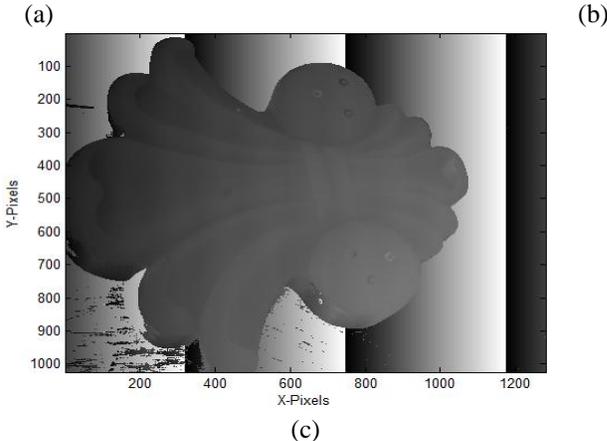
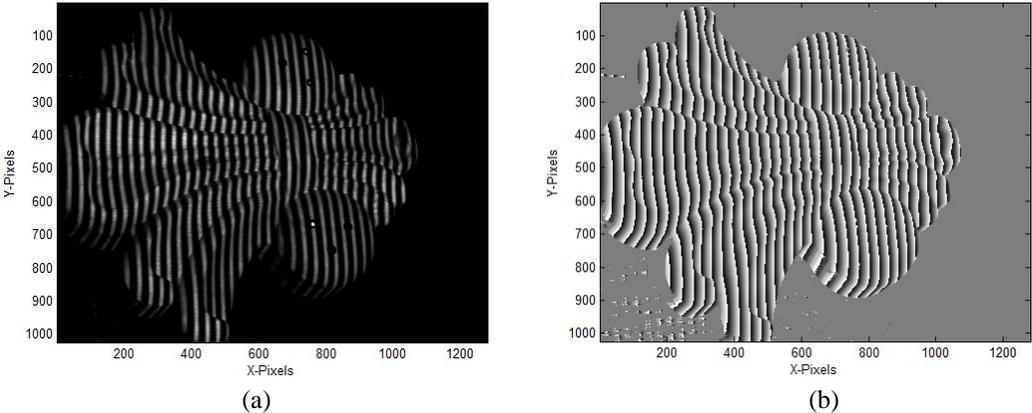


Fig 5: Relief fringe pattern remove wraps (a) A actual Relief fringe image (b) The Relief wrapped phase (c) The new phase map by shifted spectrum in frequency domain

Now changing the frequency of the projection grating on Relief and add the number of fringe is shown as Fig 6(a), the wrapped phase map is shown as Fig 6 (b), the frequency spectrum shifting result in the frequency domain is shown as Fig 6(c). In 6(c), there is still exist wraps because the spectrum shifting is not integer. To this end, the wrapped phase map 6(b) is padded with zero, the wrapped map is enlarged by 10 times, that is, 10240×12800 pixels, and the frequency accuracy is improved by 10 times, so that the non-integer shifting can be realized, the cropped image is shown as Fig 6(d). Zero padding can achieve non integer shift, the accuracy is also high, but the execution time is 8.63s, longer time and occupy larger storage space. The proposed iterative local Fourier transform method is used to estimate the fundamental frequency, and then the spectrum is shifted in the space domain, the effect is shown as Fig 6(e), and the execution time is 4.22s. From Fig 6(d) and (e), there is no significant difference in the results, but the execution time varies greatly.



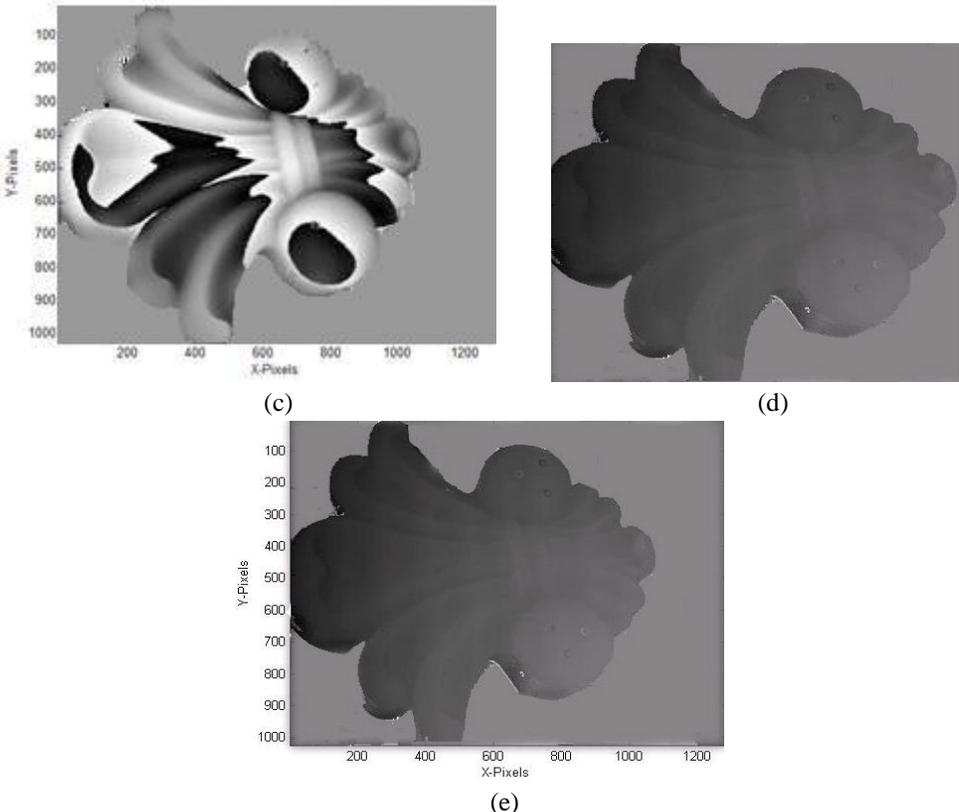


Fig 6: Preprocessed relief fringe image remove wraps (a) A Relief fringe map (b) The Relief wrapped phase image (c) The new wrapped phase by shifted spectrum in frequency domain (d) The new wrapped phase by zero-padding (e)The new wrapped phase by this proposed method

For further validate the benefit of the wraps reduction for phase unwrapping, the captured image of a nut is shown in Fig 7(a). Repeat the process above, the wrapped phase image is shown as Fig 7(b),the wraps reduction effect in frequency domain is shown as Fig 7(c),the wraps reduction effect by zero padding is shown as Fig 7(d),the proposed method result is shown in Fig 7(e). Because the fringe pattern in this example does not have any preprocessed, such as the ambient light, noise, the measured object position and phase across a larger range and so on factors influence, the effect of reducing phase wraps is unsatisfactory, subsequent phase unwrapping process is still needed.

From the above simulation and experiment, the new wrapped phase map shows that the wraps is completely eliminated or obviously reduced, and the subsequent phase unwrapping process can be avoided or accelerated and simplified.

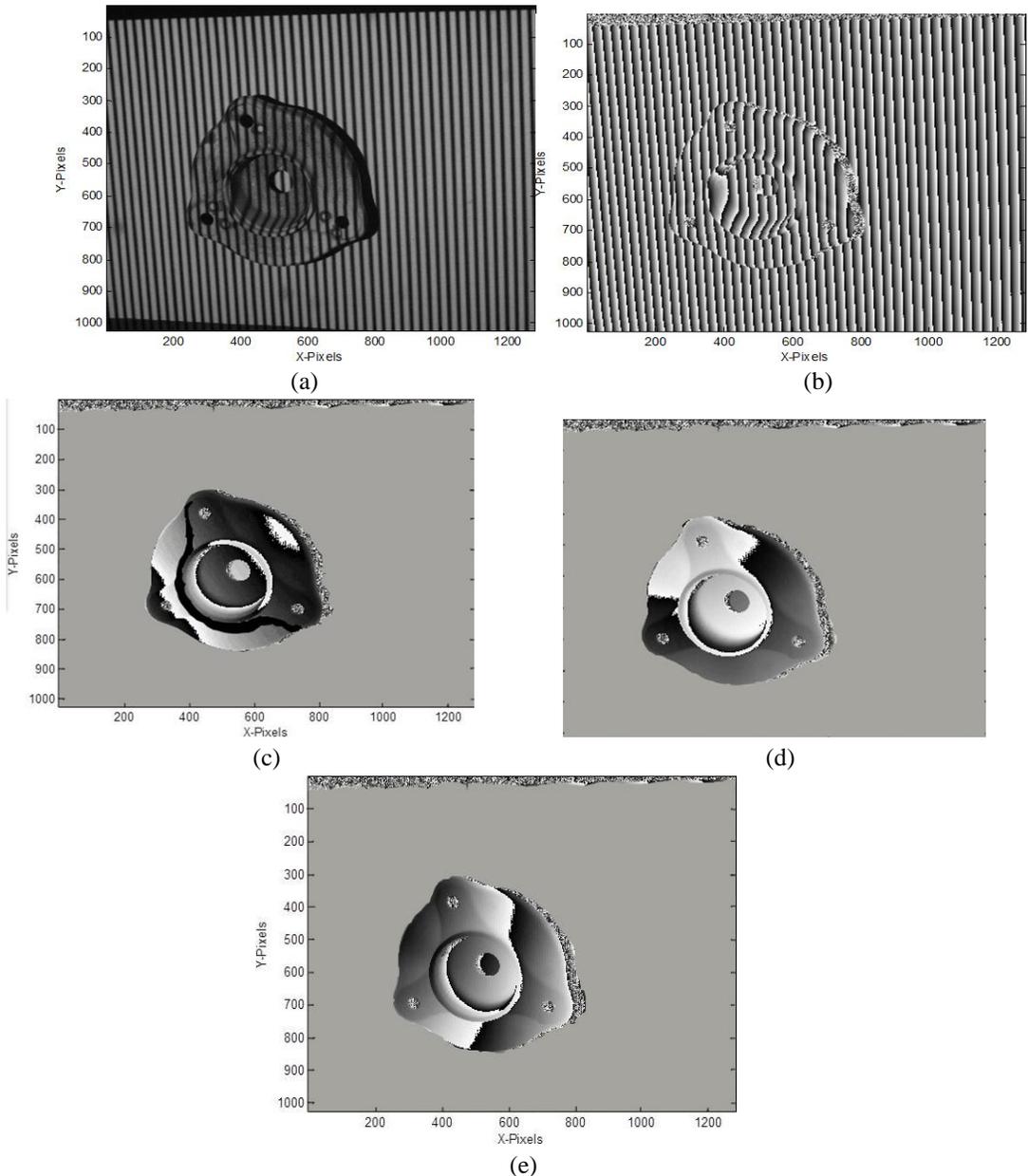


Fig 7: Nut fringe image remove wraps (a) A nut fringe image (b) The wrapped phase image of Nut (c) The new wrapped phase image by shifted spectrum in frequency domain (d) The new wrapped phase map by zero padded (e) The new wrapped phase map by this proposed method

VI. CONCLUSIONS

This paper proposes an effective algorithm to eliminate the phase wraps or decrease the number of the phase wraps of fringe images. The proposed algorithm may be used as the auxiliary of phase unwrapping step, which can avoid the phase unwrapping process or accelerate and simplify the phase unwrapping step. The proposed algorithm is compared with the existing methods based on integer shift spectrum in frequency domain and zero padding

non-integer space shift spectrum, the proposed method can achieve the accuracy of zero padding method, and save more time. Both simulations and experiments prove that the proposed method can accelerate subsequent phase unwrapping process.

In the future, we will further study the phase unwrapping accuracy and velocity problem of the new phase map behind the phase wraps are reduced by the proposed method.

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