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Multi-criteria Group Decision-Making Method Based on D Numbers

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Abstract:

Investment decision is an important aspect of multi-criterion group decision-making (MCGDM). Investment decision-making is a complicated problem because it is decided by many factors together, and there are many kinds of uncertainties in each factor. Many theories has been put forward which express and process uncertain information. And yet, these theoretical methods have some disadvantages and deficiencies. Recently, a new theoretical method, which represent and process uncertain and incomplete information, is proposed, it is called D number, and it has been tried in many fields. In this paper, we propose to transform intuitionistic interval number into D number on the basis of D number theory and intuitionistic interval number theory, and use D number to express and process uncertain information. After that, through a concrete example of investment decision, the feasibility of the new method is further verified the principle of the proposed method is easy to understand, the calculation is simple and easy to operate, more importantly, the ranking decision can be made easily. In addition, there's a lot of evidence to suggest that the proposed method can be applied to other fields.

***Keywords**: Multiple-criteria group decision-making, Numbers, Intuitionistic interval numbers, Investment decision.*

I. INTRODUCTION

In the whole investment decision-making process, there are many factors that affect and restrict the investment decision-making, and uncertainties exist in each factor, so how to quantify the factors affecting investment decision-making is an urgent problem to be solved, how to quantify the factors that affect the investment decision-making is a problem to be solved. In previous study, this problem has been studied in [1-3], and some possible situations in investment decision, such as uncertain information representation and weight determination, also have been considered. There are many multi-attribute and multi-criteria decision-making problemsin real life, investment decision-making is one of them, it is a complex problem

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because it involves various uncertainties and incomplete information. In real-life situations, the potential impact and uncertainty of investments cannot be accurately described and quantified, the investor may not know much about the investment project, or the investor himself does not have enough knowledge and understanding of the investment project. Therefore, how to express and deal with uncertain and incomplete information comprehensively, accurately and effectively is a crucial problem in investment decision-making. How to express the uncertainty problem? Relevant scholars have studied this, including rough set method, fuzzy set method, analytic hierarchy process method, probability method and Dempster-Shafer theory of evidence (D-S evidence theory) [4-10]. D-S evidence theory [11-14] is one of the most common theory, which is an important theory for dealing with uncertainty problem, which has been widely used in many research fields, such as signal processing and radar tracking.Because this theory can describe the uncertainty of the problem accurately, comprehensively and effectively. Now consider a series of events that are mutually exclusive and complementary, the basic probability evaluation has the ability to assign the reliability to any subset. The Dempster-Shafer evidence theory not only gives the method of how to express the evidence, but also gives the combination rule of the evidence, which means Dempster combination rule, Dempster combination rule is the most important part of the D-S evidence theory as far as we're concerned, this rule can be used to fuse multiple pieces of evidences. D-S evidence theory has been widely used in many research fields, including data fusion and decision-making [15-16],because it has advantages that the previous theory didn't have in representing the information of uncertainty.

However, Dempster-Shafer theory also has some drawbacks and deficiencies, because Dempster-Shafer involves a mathematical framework based on a set of assumptions, On the one hand, it requires that the elements in the framework must be independent and mutually exclusive, on the other hand, it also requires that the sum of the basic probability assignment(BPA) function must be equal to 1,these two assumptions can hardly be met at the same time.In order to overcome the shortcomings of the classical Dempster-Shafer theory, a theory called D numbers is put forward[17], which is the extension and generalization of Dempster-Shafer theory, the elements in the identification framework cannot be mutually exclusive, that is, they may have intersections, and the sum of the basic probability assignment functions may not be equal to 1, but less than 1, which is more practical and can represent various uncertainties more effectively, and appeared to represent various kinds of uncertainty effectively [11-12]. D numbers theory has been used in many fields owing to its efficiency [8,9,18-20].

Investment decision-making is a complex attribute evaluation problem, which is composed of a large number of uncertainties and incompleteness. In this paper, on the basis of D number theory and intuitionistic interval number theory[21], we proposes to transform intuitionistic interval number into d number in this paper, and use D number to express and deal with uncertain information, and the numerical example shows that the method is feasible. The principle of the proposed method is easy to understand, the calculation is simple and easy to operate, more importantly, and the ranking decision can be made easily.

The structure of this article is as follows. In Section 2, we give some preliminaries, including D numbers and intuitionistic interval numbers. In Section3, a new method about multi-criteria group decision-making is proposed, we first transform intuitionistic interval number into D number, then we use D number to express and process the uncertain information, finally we can sort the decision. In Section 4, we give a numerical example,and some conclusions and discussions will be presented in Section 5.

II. PRELIMINARIES

In this section, we will introduce some preliminary knowledge, which will be used later sections.

2.1 D Numbers

Definition 2.1 [17]: Let Ω be a finite set,and $\Omega \neq \Phi$, D numbers is a correspondence relation, $D: 2^{\Omega} \rightarrow [0,1]$, such that:

$$\sum_{A \subseteq \Omega} D(A) \leq 1 \text{ and } D(\Phi) = 0$$

where Φ is an empty set, in the meantime $A \subseteq \Omega$. It is worth noting here that, on the one hand, the elements in the set Ω of D numbers could have crossed paths,on the other hand,the sum of the estimates may not equal to 1, and in some cases may be less than 1 in D numbers.

Let's assume the assessment score is in interval $[0,100]$, the evaluation given by the judges under the framework of Dempster-Shafer theory identification is as follows:

$$m(a_1) = 0.4,$$

$$m(a_2) = 0.5,$$

$$m(a_1, a_2, a_3) = 0.1,$$

where $a_1 = [1,25]$, $a_2 = [26,73]$, $a_3 = [74,100]$.It's worth noting that, $a_1 \cap a_2 = \Phi$, $a_2 \cap a_3 = \Phi$, and (a_1, a_2, a_3) is the framework for the evidence of theory. $\sum_i m_i = 1$,that means it's complete.

Meanwhile, another judgesunder the framework of D numbers is given by D numbers as follows:

$$D(b_1) = 0.4,$$

$$D(b_2) = 0.3,$$

$$D(b_1, b_2, b_3) = 0.1,$$

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where $b_1=[1,40]$, $b_2=[35,70]$, $b_3=[65,100]$. It's worth noting that, $\{b_1,b_2,b_3\}$ are not mutually exclusive, in other words, $b_1 \cap b_2 \neq \Phi$, $b_2 \cap b_3 \neq \Phi$. In the meantime, $\sum_i D_i = 0.8 < 1$, this is called incomplete information if sum is less than 1.

Definition 2.2 [17]: Given a discrete set $\Omega=\{b_1,b_2,b_3\}$,in which $b_i \in R$, if $i \neq j$,then $b_i \neq b_j$, $\forall\ v_i \geq 0$, $\sum_{i=1}^{n} v_i \leq 1$. So the D numbers can be expressed in the following way:

$$D(b_1)=v_1,$$
$$D(b_2)=v_2,$$
$$D(b_3)=v_3,$$
$$\cdots\cdots$$
$$D(b_n)=v_n,$$

it can also be expressed simply as:

$$D=\{(b_1,v_1),(b_2,v_2),(b_3,v_3),\cdots,(b_n,v_n)\}.$$

Definition2.3 [17]:Let D_1 and D_2 be two D numbers,

$$D_1=\{(b_1^1,v_1^1),(b_2^1,v_2^1),(b_3^1,v_3^1),\cdots,(b_n^1,v_n^1)\},$$
$$D_2=\{(b_1^2,v_1^2),(b_2^2,v_2^2),(b_3^2,v_3^2),\cdots,(b_n^2,v_n^2)\},$$

Mark the combination of D_1 and D_2 as $D_1 \oplus D_2$, define it as follows:

$$b=\frac{b_i^1+b_j^2}{2},$$

$$v=\frac{v_i^1+v_j^2}{2C},$$

where,

$$C=\begin{cases} \sum_{j=1}^{m}\sum_{i=1}^{n}\frac{v_i^1+v_j^2}{2}, & if\ \sum_{i=1}^{n}v_i=1\ and\ \sum_{j=1}^{m}v_j=1, \\ \sum_{j=1}^{m}\sum_{i=1}^{n}\frac{v_i^1+v_j^2}{2}+\sum_{j=1}^{m}\frac{v_c^1+v_j^2}{2}, & if\ \sum_{i=1}^{n}v_i<1\ and\ \sum_{j=1}^{m}v_j=1, \\ \sum_{j=1}^{m}\sum_{i=1}^{n}\frac{v_i^1+v_j^2}{2}+\sum_{i=1}^{n}\frac{v_i^1+v_c^2}{2}, & if\ \sum_{i=1}^{n}v_i=1\ and\ \sum_{j=1}^{m}v_j<1, \\ \sum_{j=1}^{m}\sum_{i=1}^{n}\frac{v_i^1+v_j^2}{2}+\sum_{j=1}^{m}\frac{v_c^1+v_j^2}{2}+\sum_{i=1}^{n}\frac{v_i^1+v_c^2}{2}, & if\ \sum_{i=1}^{n}v_i<1\ and\ \sum_{j=1}^{m}v_j<1, \end{cases}$$

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in which, $v_c^1 = 1 - \sum_{i=1}^{n} v_i^1$ and $v_c^2 = 1 - \sum_{j=1}^{m} v_j^2$,the superscript in the above equation cannot be treated as an exponent, but rather as the order of D numbers.

Definition 2.4 [17]: Given a D numbers $D = \{(b_1, v_1), (b_2, v_2), (b_3, v_3), \cdots, (b_n, v_n)\}$, the formula for the overall assessment is:

$$I(D) = \sum_{i=1}^{n} (b_i v_i) .$$

2.2 Intuitionistic Interval Numbers

Definition 2.5 [21]: Let a is an interval number, then we can defined as follows:

$$a = [a^L, a^U] = \{x \mid a^L \le x \le a^U\} .$$

in which a^L is called lower bounds of a , a^U is called upper bounds of a . Particularly, if $a^L = a^U$, then a is a concrete real number.

Definition 2.6 [21]: Let $\bar{a}$ is an intuitionistic interval number, it takes the form as follows:

$$\bar{a} = \langle [a^L, a^U]; u_{\bar{a}}, v_{\bar{a}} \rangle ,$$

where $[a^L, a^U]$ is called the interval part of $\bar{a}$, accordingly, the intuitive part of $\bar{a}$ is $\langle u_{\bar{a}}, v_{\bar{a}} \rangle$.

We define the membership function $u_{\bar{a}}(x)$ of $\bar{a}$ as:

$$u_{\bar{a}}(x) = \begin{cases} u_{\bar{a}}, & a^L \le x \le a^U, \\ 0, & others, \end{cases}$$

and we define the non-membership function $v_{\bar{a}}(x)$ of $\bar{a}$ as:

$$v_{\bar{a}}(x) = \begin{cases} v_{\bar{a}}, & a^L \le x \le a^U, \\ 1, & others, \end{cases}$$

where $0 \le u_{\bar{a}} \le 1$, $0 \le v_{\bar{a}} \le 1$, $u_{\bar{a}} + v_{\bar{a}} \le 1$, the hesitation function $\pi_{\bar{a}} = 1 - u_{\bar{a}} - v_{\bar{a}}$. Thus, the closed interval $[u_{\bar{a}}, 1 - v_{\bar{a}}]$ is called the membership degree of $\bar{a}$, Particularly, if $u_{\bar{a}} = 1$, $v_{\bar{a}} = 0$, then $\bar{a}$ is reduced to an interval number, which means $\bar{a} = [a^L, a^U] = a$.

Example1: Let $\bar{a} = \langle [4,6]; 0.5, 0.4 \rangle$ is an intuitionistic interval number, where 0.5 is the membership of $\bar{a}$, and 0.4 is the non-membership of $\bar{a}$.

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III. PROPOSED METHOD

According to the introduction in Section 2, we define a new method, aiming to convert intuitionistic interval number to D numbers.

Definition3.1 Given an intuitionistic interval number $\bar{a} = \langle [a^L, a^U]; u_{\bar{a}}, v_{\bar{a}} \rangle$, we define D number $D = \{(b, v)\}$, where $b = \frac{a^L + a^U}{2}$, $v = u_{\bar{a}}$.

Example2: Given an intuitionistic interval number $\bar{a} = \langle [4,6]; 0.5, 0.4 \rangle$, the corresponding D number of which is $D = \{(\frac{4+6}{2}, 0.5)\} = \{(5, 0.5)\}$.

Definition 3.2 Let $\bar{a}_i = \langle [a_i^L, a_i^U]; u_{\bar{a}_i}, v_{\bar{a}_i} \rangle (i = 1,2,\cdots,n)$ be a group of intuitionistic interval numbers. If $\sum_{i=1}^{n} u_{\bar{a}_i} \le 1$, we define D numbers $D = \{(b_i, v_i)\}$, where $b_i = \frac{a_i^L + a_i^U}{2}$, $v_i = u_{\bar{a}_i}$. If $\sum_{i=1}^{n} u_{\bar{a}_i} > 1$, we define D numbers $D = \{(b_i, v_i)\}$, where $b_i = \frac{a_i^L + a_i^U}{2}$, $v_i = \frac{u_{\bar{a}_i}}{\sum_{i=1}^{n} u_{\bar{a}_i}}$.

Example3: There is a group of intuitionistic interval numbers: $\bar{a}_1 = \langle [4,6]; 0.5, 0.4 \rangle$, $\bar{a}_2 = \langle [6,8]; 0.6, 0.3 \rangle$, $\bar{a}_3 = \langle [6,9]; 0.5, 0.3 \rangle$, $\bar{a}_4 = \langle [3,5]; 0.5, 0.4 \rangle$.

According to $\sum_{i=1}^{4} u_{\bar{a}_i} = 0.5 + 0.6 + 0.5 + 0.5 = 2.1 > 1$, we calculate the corresponding D numbers:

$$D = \{(\frac{4+6}{2}, \frac{0.5}{2.1}), (\frac{6+8}{2}, \frac{0.6}{2.1}), (\frac{6+9}{2}, \frac{0.5}{2.1}), (\frac{3+5}{2}, \frac{0.5}{2.1})\}.$$

IV. AN EXAMPLE

The degree to which the code is standardized is a major consideration in judging a programmer's work. How to evaluate the programmer's code is good code is a comprehensive evaluation of the problem? In general, good code should have the following qualities: consistency, readability, and maintainability and so on, when programmers write code with these qualities, it is easier for programs to be shared, developed, and perfected among programmers or teams. For example, a software company needs to assign one of its three programmers to build a customer relationship with another company, in order to establish a system which is about management information. How does the company dispatch candidates? When selecting a candidate, three executives $d_k (k = 1,2,3)$ will assess the level of standardization of the code written by the three candidates $A_i (i = 1,2,3)$ based on their level of standardization. The subjective weight of the first executive is 0.5, the second one is 0.3 and

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that of the third one is 0.2. The attitude factor of the executives are all 0.5[21]. This problem is a typical multi-attribute decision-making problem. For the convenience of operation, we will evaluate three candidates from the following four benefit criteria:

(1) C_1: Whetherthe class name is concise or not?

(2) C_2: Whether the note is clear or not?

(3) C_3: Whether it is sharp or not?

(4) C_4:Whether the indentation character is standard or not ?

The weights of this criterias are as follows:

$\langle[0.15,0.25];0.6,0.1\rangle,\langle[0.2,0.3];0.5,0.4\rangle,\langle[0.25,0.4];0.6,0.2\rangle,\langle[0.2,0.25];0.7,0.2\rangle$.

Tables 1 to 3 are the decision matrices of three executives'evaluations of three candidates from four benefit criteria

TABLE I. The ambiguous decision matrix made by executive d_1

	C_1	C_2	C_3	C_4
A_1	$\langle[4,6];0.5,0.4\rangle$	$\langle[6,8];0.6,0.3\rangle$	$\langle[6,9];0.5,0.3\rangle$	$\langle[3,5];0.5,0.4\rangle$
A_2	$\langle[4,8];0.7,0.2\rangle$	$\langle[5,7];0.5,0.2\rangle$	$\langle[7,9];0.3,0.2\rangle$	$\langle[3,6];0.4,0.2\rangle$
A_3	$\langle[5,7];0.5,0.3\rangle$	$\langle[6,7];0.6,0.3\rangle$	$\langle[6,8];0.6,0.2\rangle$	$\langle[4,7];0.5,0.3\rangle$

TABLE II. The ambiguous decision matrix made by executive d_2

	C_1	C_2	C_3	C_4
A_1	$\langle[5,7];0.7,0.2\rangle$	$\langle[7,8];0.8,0.1\rangle$	$\langle[5,8];0.8,0.1\rangle$	$\langle[2,5];0.7,0.1\rangle$
A_2	$\langle[5,8];0.6,0.2\rangle$	$\langle[6,7];0.7,0.1\rangle$	$\langle[6,8];0.8,0.1\rangle$	$\langle[3,6];0.7,0.1\rangle$
A_3	$\langle[6,8];0.7,0.1\rangle$	$\langle[7,9];0.7,0.2\rangle$	$\langle[5,9];0.7,0.2\rangle$	$\langle[3,5];0.8,0.1\rangle$

TABLE III. The ambiguous decision matrix made by executive d_3

	C_1	C_2	C_3	C_4
A_1	$\langle[4,7];0.6,0.3\rangle$	$\langle[7,9];0.5,0.4\rangle$	$\langle[6,7];0.6,0.2\rangle$	$\langle[3,5];0.7,0.2\rangle$
A_2	$\langle[5,8];0.6,0.1\rangle$	$\langle[7,8];0.5,0.2\rangle$	$\langle[6,9];0.7,0.2\rangle$	$\langle[4,6];0.6,0.2\rangle$
A_3	$\langle[5,7];0.8,0.1\rangle$	$\langle[6,8];0.7,0.1\rangle$	$\langle[7,8];0.6,0.3\rangle$	$\langle[4,5];0.8,0.1\rangle$

Next, we will solve this practical problem one after the other by applying the proposed new method.

Step 1: Convert intuitionistic interval numbers to D numbers.

According to Definition 3.2, we convert intuitionistic interval number to D numbers.

$$D_{A_1}^{d_1}=\left\{(\frac{4+6}{2},\frac{0.5}{2.1}),(\frac{6+8}{2},\frac{0.6}{2.1}),(\frac{6+9}{2},\frac{0.5}{2.1}),(\frac{3+5}{2},\frac{0.5}{2.1})\right\};$$

$$D_{A_2}^{d_1}=\left\{(\frac{4+8}{2},\frac{0.7}{1.9}),(\frac{5+7}{2},\frac{0.5}{1.9}),(\frac{7+9}{2},\frac{0.3}{1.9}),(\frac{3+6}{2},\frac{0.4}{1.9})\right\};$$

$$D_{A_3}^{d_1}=\left\{(\frac{5+7}{2},\frac{0.5}{2.2}),(\frac{6+7}{2},\frac{0.6}{2.2}),(\frac{6+8}{2},\frac{0.6}{2.2}),(\frac{4+7}{2},\frac{0.5}{2.2})\right\}.$$

$$D_{A_1}^{d_2}=\left\{(\frac{5+7}{2},\frac{0.7}{3}),(\frac{7+8}{2},\frac{0.8}{3}),(\frac{5+8}{2},\frac{0.8}{3}),(\frac{2+5}{2},\frac{0.7}{3})\right\};$$

$$D_{A_2}^{d_2}=\left\{(\frac{5+8}{2},\frac{0.6}{2.8}),(\frac{6+7}{2},\frac{0.7}{2.8}),(\frac{6+8}{2},\frac{0.8}{2.8}),(\frac{3+6}{2},\frac{0.7}{2.8})\right\};$$

$$D_{A_3}^{d_2}=\left\{(\frac{6+8}{2},\frac{0.7}{2.9}),(\frac{7+9}{2},\frac{0.7}{2.9}),(\frac{5+9}{2},\frac{0.7}{2.9}),(\frac{3+5}{2},\frac{0.8}{2.9})\right\}.$$

$$D_{A_1}^{d_3}=\left\{(\frac{4+7}{2},\frac{0.6}{2.4}),(\frac{7+9}{2},\frac{0.5}{2.4}),(\frac{6+7}{2},\frac{0.6}{2.4}),(\frac{3+5}{2},\frac{0.7}{2.4})\right\};$$

$$D_{A_2}^{d_3}=\left\{(\frac{5+8}{2},\frac{0.6}{2.4}),(\frac{7+8}{2},\frac{0.5}{2.4}),(\frac{6+9}{2},\frac{0.7}{2.4}),(\frac{4+6}{2},\frac{0.6}{2.4})\right\};$$

$$D_{A_3}^{d_3}=\left\{(\frac{5+7}{2},\frac{0.8}{2.9}),(\frac{6+8}{2},\frac{0.7}{2.9}),(\frac{7+8}{2},\frac{0.6}{2.9}),(\frac{4+5}{2},\frac{0.8}{2.9})\right\}.$$

Step 2: D number combination

According to the Definition 2.3,we combine the D numbers.

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$D_{A_1} = D_{A_1}^{d_1} \oplus D_{A_1}^{d_2} \oplus D_{A_1}^{d_3} = \{$(5.5000, 0.2429), (6.0000, 0.2548), (6.1250, 0.2429), (5.2500, 0.2429), (5.8750, 0.2512), (6.3750, 0.2631), (6.5000, 0.2512), (5.6250, 0.2512), (5.6250, 0.2512), (6.1250, 0.2631), (6.2500, 0.2512), (5.3750, 0.2512), (4.8750, 0.2429), (5.3750, 0.2548), (5.5000,0.2429),(4.6250,0.2429), (6.7500, 0.2220), (7.2500, 0.2339), (7.3750, 0.2220), (6.5000, 0.2220), (7.1250,0.2304), (7.6250,0.2423), (7.7500, 0.2304), (6.8750, 0.2304), (6.8750, 0.2304), (7.3750, 0.2423), (7.5000, 0.2304), (6.6250, 0.2304), (6.1250, 0.2220), (6.6250, 0.2339), (6.7500, 0.2220), (5.8750, 0.2220), (6.0000, 0.2429), (6.5000, 0.2548), (6.6250, 0.2429), (5.7500, 0.2429), (6.3750,0.2512),(6.8750,0.2631),(7.0000,0.2512),(6.1250,0.2512), (6.1250,0.2512),(6.6250, 0.2631), (6.7500, 0.2512), (5.8750, 0.2512), (5.3750, 0.2429), (5.8750, 0.2548), (6.0000, 0.2429), (5.1250, 0.2429), (4.7500, 0.2637), (5.2500, 0.2756), (5.3750, 0.2637), (4.5000, 0.2637), (5.1250, 0.2720), (5.6250, 0.2839), (5.7500, 0.2720), (4.8750, 0.2720), (4.8750, 0.2720), (5.3750, 0.2839), (5.5000, 0.2720), (4.6250, 0.2720), (4.1250, 0.2637), (4.6250, 0.2756), (4.7500, 0.2637), (3.8750, 0.2637)$\}$.

$D_{A_2} = D_{A_2}^{d_1} \oplus D_{A_2}^{d_2} \oplus D_{A_2}^{d_3} = \{$(6.3750,0.2707),(6.3750,0.2444),(6.8750,0.2180),(6.0000,0.2312), (6.3750,0.2796),(6.3750,0.2533),(6.8750,0.2270),(6.0000,0.2401),(6.5000,0.2885),(6.5000,0.2622), (7.0000,0.2359),(6.1250,0.2491),(5.8750,0.2796),(5.8750,0.2533),(6.3750,0.2270),(5.5000,0.2401), (6.8750,0.2498),(6.8750,0.2235),(7.3750,0.1972),(6.5000,0.2104),(6.8750,0.2588),(6.8750,0.2325), (7.3750,0.2061),(6.5000,0.2193),(7.0000,0.2677),(7.0000,0.2414),(7.5000,0.2151),(6.6250,0.2282), (6.3750,0.2588),(6.3750,0.2325),(6.8750,0.2061),(6.0000,0.2193),(6.8750,0.2915),(6.8750,0.2652), (7.3750,0.2389),(6.5000,0.2520),(6.8750,0.3004),(6.8750,0.2741),(7.3750,0.2478),(6.5000,0.2610), (7.0000,0.3094),(7.0000,0.2831),(7.5000,0.2567),(6.6250,0.2699),(6.3750,0.3004),(6.3750,0.2741), (6.8750,0.2478),(6.0000,0.2610),(5.6250,0.2707),(5.6250,0.2444),(6.1250,0.2180),(5.2500,0.2312), (5.6250,0.2796),(5.6250,0.2533),(6.1250,0.2270),(5.2500,0.2401),(5.7500,0.2885),(5.7500,0.2622), (6.2500,0.2359),(5.3750,0.2491),(5.1250,0.2796),(5.1250,0.2533),(5.6250,0.2270),(4.7500,0.2401)$\}$.

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$D_{A_3}=D_{A_3}^{d_1}\oplus D_{A_3}^{d_2}\oplus D_{A_3}^{d_3}=$ {(6.2500,0.2551),(6.3750,0.2665),(6.5000,0.2665),(6.1250,0.2551),
(6.5000,0.2551),(6.6250,0.2665),(6.7500,0.2665),(6.3750,0.2551),(6.2500,0.2551),(6.3750,0.2665),
(6.5000,0.2665),(6.1250,0.2551),(5.5000,0.2637),(5.6250,0.2751),(5.7500,0.2751),(5.3750,0.2637),
(6.7500,0.2379),(6.8750,0.2492),(7.0000,0.2492),(6.6250,0.2379),(7.0000,0.2379),(7.1250,0.2492),
(7.2500,0.2492),(6.8750,0.2379),(6.7500,0.2379),(6.8750,0.2492),(7.0000,0.2492),(6.6250,0.2379),
(6.0000,0.2465),(6.1250,0.2578),(6.2500,0.2578),(5.8750,0.2465),(7.0000,0.2206),(7.1250,0.2320),
(7.2500,0.2320),(6.8750,0.2206),(7.2500,0.2206),(7.3750,0.2320),(7.5000,0.2320),(7.1250,0.2206),
(7.0000,0.2206),(7.1250,0.2320),(7.2500,0.2320),(6.8750,0.2206),(6.2500,0.2292),(6.3750,0.2406),
(6.5000,0.2406),(6.1250,0.2292),(5.5000,0.2551),(5.6250,0.2665),(5.7500,0.2665),(5.3750,0.2551),
(5.7500,0.2551),(5.8750,0.2665),(6.0000,0.2665),(5.6250,0.2551),(5.5000,0.2551),(5.6250,0.2665),
(5.7500,0.2665),(5.3750,0.2551),(4.7500,0.2637),(4.8750,0.2751),(5.0000,0.2751),(4.6250,0.2637)}.

Step 3: Calculate D numbers of the overall assessment.

According to Definition 2.4, we calculate D numbers the overall assessment:

$$I(D_{A_1})=\sum_{i=1}^{64}(b_i v_i)=94.4619,$$

$$I(D_{A_2})=\sum_{i=1}^{64}(b_i v_i)=101.8929,$$

$$I(D_{A_3})=\sum_{i=1}^{64}(b_i v_i)=100.5110.$$

Step 4: Rank the overall assessment of D numbers:

$$I(D_{A_2})>I(D_{A_3})>I(D_{A_1}),$$

So the preference order is $A_2>A_3>A_1$. Therefore, we choose the second candidates, the selection results are consistent with those in reference[21].

V. CONCLUSIONS

This paper introduces the definition and Operation Law of D numbers and intuitionistic interval numbers. We propose to transform intuitionistic interval number into D number on the basis of D number theory and intuitionistic interval number theory in this paper, thus, we use D number to express and deal with uncertain information. Moreover, an example of investment decision-making shows that the new method is practicable. The rationale behind this approach is easy to understand, it's even more convenient that the calculation is simple and easy to operate. Consequently, the ranking decision can be easily made.

When we make a decision on a number of options, because the decision-making process is quite complex, and the factors affecting the choice may also be fuzzy, the decision-maker is

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often unable to give an accurate judgment.Usually when this uncertainty is ignored, D numbers could solve this kind of problem nicely, at the same time, the proposed approach fits with people's thinking patterns.Since D numbers theory has many advantages, we can make the decision-making process more practical,and reflects a real mind-set more accurately when we apply D numbers theory to figure out how to make decisions.In futureresearch, we hope that the new method proposed in this article, which is using D numbers to solve decision problems, other decision-making problems in different fields of study can also be applied, and D numbers will be developed and improved, the goal is to solve some problems perfectly, such as uncertainty and incompleteness.

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