# Research on Electrified Railway Harmonics Detection Based on Sliding-window Iterative DFT Algorithm

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### Abstract:

Harmonic detection is a key link in the electrification railway harmonic suppression. Detection precision directly affects the effect of filter. And only accurately detect the harmonic, reasonable inhibition can be conducted. This paper analyzes the shortcomings of the present stage of harmonic current detection methods, such as a large amount of calculation and requirement low pass filtering link. For electrified railway harmonic characteristics in accordance with the active power filter harmonic detection real-time, accuracy requirements, an iterative discrete Fourier transform method for sliding windows is proposed. Simulation results show that the iterative discrete Fourier method of sliding-window for harmonic analysis can accurately detect traction's load current base wave and every harmonic current component. In the case of traction load mutations, the algorithm can also in a power frequency cycle accurately track changes of load current and the harmonic detection current real-time.

Keywords: DFT, Electrification railway, Harmonic detection, Power quality.

### I. INTRODUCTION

Electrified railway harmonic suppression not only improves electrification railway traction substation power supply reliability, but also on the local power grid safety and reliability of the operation has a very important role [1]. Harmonic detection is the key link of electrification railway harmonic suppression, precision directly affects the effect of filter. Analog filter to circuit element parameter is very sensitive. And errors of phase and amplitude are relatively large [2]. Fast Fourier algorithm has although high detection accuracy, but easy to produce the spectrum leakage, cause the detection error, and the large amount of calculation, real time is bad [3]. The low-pass filter in the harmonic analysis method of instantaneous reactive power has a certain delay [4]. The wavelet transform, neural network, adaptive detection method and other intelligent algorithm, with advantages of high-precision, good response, but large amount of calculation, engineering realization is more difficult [5]. Method of iterative discrete Fourier transform (DFT) for sliding-window is applied to active power filter harmonic suppression. It

has very high calculation accuracy.

#### **II. DETECTION PRINCIPLE OF SLIDING-WINDOW ITERATION DFT METHOD**

Supposing the current  $i(\omega t)$  is a non-sinusoidal signal and satisfies the Dirichlet conditions, that it will be expressed in the following series as (1).

$$i(\omega t) = a_0 + \sum_{n=1}^{\infty} \left[ a_n \cos(n\omega t) + b_n \sin(n\omega t) \right] (1)$$

Among them,

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} i(\omega t) d(\omega t)$$
$$a_n = \frac{1}{\pi} \int_0^{2\pi} i(\omega t) \cos(n\omega t) d(\omega t)$$
$$b_n = \frac{1}{\pi} \int_0^{2\pi} i(\omega t) \sin(n\omega t) d(\omega t)$$

Combining  $e^{jn\omega t} = \cos(n\omega t) + j\sin(n\omega t)$  and  $e^{-jn\omega t} = \cos(n\omega t) - j\sin(n\omega t)$  with formula (1),  $i(\omega t)$  is derived as equation (2).

$$i(\omega t) = a_0 + \sum_{n=1}^{\infty} \left(\frac{a_n - jb_n}{2}e^{jn\omega t} + \frac{a_n + jb_n}{2}e^{-jn\omega t}\right) (2)$$

Extending the range of n to the entire field of real numbers, then  $sin(n\omega t)$  and  $cos(n\omega t)$  are odd function and even function of n respectively. The  $a_0$  can be derived as equation (3).

$$a_0 = (a_n/2)_{n=0} = (\frac{a_n - jb_n}{2}e^{jn\omega t})_{n=0}$$
(3)

$$i(\omega t) = a_0 + \sum_{n=1}^{\infty} \left(\frac{a_n - jb_n}{2}e^{jn\omega t} + \frac{a_n + jb_n}{2}e^{-jn\omega t}\right)$$
  
=  $\left(\frac{a_n - jb_n}{2}e^{jn\omega t}\right)_{n=0} + \sum_{n=1}^{\infty} \frac{a_n - jb_n}{2}e^{jn\omega t} + \sum_{n=-1}^{-\infty} \frac{a_n - jb_n}{2}e^{-jn\omega t}$  (4)  
=  $\sum_{n=-\infty}^{\infty} \frac{a_n - jb_n}{2}e^{jn\omega t}$ 

Supposing  $F_n$  is a variable, namely

$$\dot{F}_n = (a_n - jb_n)/2. \tag{5}$$

So then

$$\dot{F}_n = \frac{1}{2} \sqrt{a_n^2 + b_n^2} e^{j\theta_n} = \frac{1}{2} A_n e^{j(\varphi_n - 90^\circ)}.$$
(6)

The amplitude of nth harmonic is equal to  $A_n/2$ . The argument is  $\theta_n = \arctan(-b_n/a_n) = \varphi_n - 90^\circ$ . As  $\dot{F}_n$  and  $\dot{F}_{-n}$  are conjugate complex numbers, nth harmonic can be derive as (7) when extending n from  $-\infty$  to  $+\infty$ .

$$\dot{F}_{n} e^{jn\omega t} + \dot{F}_{-n} e^{-jn\omega t} = \frac{A_{n}}{2} e^{j\theta_{n}} e^{jn\omega t} + \frac{A_{n}}{2} e^{-j\theta_{n}} e^{-jn\omega t}$$

$$= A_{n} \cos(n\omega t + \theta_{n}) = A_{n} \sin(n\omega t + \varphi_{n})$$
(7)

In the engineering practices of electrified railway, the distorted current waveform of traction power supply is unknown [6]. So it are unable to equal it with a certain resolution function first and then calculate in. It tend to organize interval sampling by adding N evenly spaced sampling points within the period cycle T for periodic current waveform in continuous time, then transform sampling values into a series of digital signal and carry out quick Fourier analysis with computer [7]. For the Fourier series of continuous function, it must be transformed into approximate Fourier series in discrete sequences. It means that continuous function i(t) can be transformed into discrete series  $\{i_k\}$ .

Combining  $n\omega_0 t = n \frac{2\pi}{T} \times \frac{T}{N} k = \frac{2\pi k}{N} n$  and  $dt = \Delta t = \tau = T/N$  with  $F_n$ , the following

equations can be obtained.

$$a_n = \frac{2}{T} \sum_{k=0}^{N-1} i_k \cos \frac{2\pi k}{N} n \times \frac{T}{N} = \frac{2}{N} \sum_{k=0}^{N-1} i_k \cos \frac{2\pi k}{N} n, \quad n = 1, 2, \dots, N-1.$$
(8)

$$b_n = \frac{2}{T} \sum_{k=0}^{N-1} i_k \sin \frac{2\pi k}{N} n \times \frac{T}{N} = \frac{2}{N} \sum_{k=0}^{N-1} i_k \sin \frac{2\pi k}{N} n, \quad n = 1, 2, \dots, N-1.$$
(9)

The discrete Fourier transform can be derived as (10).

$$I_n = \frac{1}{N} \sum_{k=0}^{N-1} i_k e^{-j\frac{2\pi}{N}kn} \qquad n = 1, 2, \dots, N-1$$
(10)

To improve the effectiveness of the detection, DFT is introduced. The calculations of  $a_n$  and  $b_n$  can be measured from the latest sampling point instead of the point of k = 0. It speeds up the renewal rate of data and reduces the amount of calculation. The improved  $a_n$  and  $b_n$  are indicted as (11) and (12).

$$a_n = \frac{2}{N} \sum_{k=N_{current}}^{N_{current}} i(k\tau) \cos(\frac{2\pi k}{N} k\tau) (11)$$
$$b_n = \frac{2}{N} \sum_{k=N_{current}}^{N_{current}} i(k\tau) \sin(\frac{2\pi k}{N} k\tau) (12)$$

Among them,  $N_{current}$  represents the latest sampling point. The  $i(k\tau)$  represents the sampling value in k cycles before  $N_{current}$ .

From above calculations, supposing the fundamental current component  $i_1(t\tau) = a_1 \cos(\omega t\tau) + b_1 \sin(\omega t\tau)$ , simplify the sum of  $a_1$  and  $b_1$  as equation (13) and equation (14).

$$\sum_{k=N_{current}}^{N_{current}-N+1} i(k\tau) \cos\left(\frac{2\pi k}{N} k\tau\right) = \sum_{k=N_{current}-1}^{N_{current}-N} i(k\tau) \cos\left(\frac{2\pi k}{N} k\tau\right)$$
$$-i[(N_{current}-N)\tau] \cos\left[\frac{2\pi k}{N} (N_{current}-N)\tau\right] \qquad (13)$$
$$+i(N_{current}\tau) \cos\left(\frac{2\pi k}{N} N_{current}\tau\right)$$
$$\sum_{k=N_{current}}^{N_{current}-N+1} i(k\tau) \sin\left(\frac{2\pi k}{N} k\tau\right) = \sum_{k=N_{current}-1}^{N_{current}-N} i(k\tau) \sin\left(\frac{2\pi k}{N} k\tau\right)$$
$$-i[(N_{current}-N)\tau] \sin\left[\frac{2\pi k}{N} (N_{current}-N)\tau\right] \qquad (14)$$
$$+i(N_{current}\tau) \sin\left(\frac{2\pi k}{N} N_{current}\tau\right)$$

Equation (13) and equation (14) represent the fundamental current component at the point of t. Subtract the certain value before  $N_{current}$  from the sum of sampling value in k cycles  $N_{current}$  and plus the value of  $N_{current}$ . Its result is equal to the fundamental current component at the point of t. The iteration is achieved by replacing the old total value of the sampling points with the new total value. The update speed of the data is accelerated and the accuracy of current detection is greatly improved [8].

In electrified railway, the main components of harmonics are the third, fifth and seventh odd harmonics. Current quality of electrified railway will be a qualitative leap if these harmonics were filtered. If 3<sup>rd</sup>, 5<sup>th</sup> and 7<sup>th</sup> harmonics are collected out respectively in the same time, it will provide convenience to the design of harmonic suppression equipment based on active power filters (APF). It is easy to detect single harmonic distorted current by using the sliding-window iterative algorithm. Take detection 3<sup>rd</sup> harmonic for example as (15).

 $i_3(t\tau) = A_3 \cos(3\omega t\tau) + B_3 \sin(3\omega t\tau) (15)$ 

Among them,

$$A_{3} = \left[\frac{2}{N} \sum_{k=N_{current}}^{N_{current}-N+1} u(k\tau) \cos(3\omega k\tau)\right]$$
$$B_{3} = \left[\frac{2}{N} \sum_{k=N_{current}}^{N_{current}-N+1} u(k\tau) \sin(3\omega k\tau)\right]$$

#### **III. HARMONIC ANALYSIS OF MOVING WINDOW ITERATION DFT METHOD**

The distortion current of electric locomotive model is loaded to workspace in the MATLAB environment. Equation (13) and equation (14) are able to collect 3rd, 5th and 7th harmonic

respectively in the same time. Defining the base wave sampling frequency is equal to6400Hz, then Nth harmonic sampling frequency is 6400/n Hz and the number of sampling points is  $N = 1/[n \times f_s \times (n/6400)]$ . The simulation waves are shown in Fig 1 to Fig 4.





Above figures represent that the 3<sup>rd</sup>, 5<sup>th</sup> and 7<sup>th</sup> harmonic components are detected completely and accurately according tomoving-window iterationdiscrete Fourier transformmethod. The detection method with less computation reports small errors and capability for real time. The result can be regarded as command current in the harmonic suppression of electric railway. And make active power filters (APF) can suppress the traction

over the harmonic wave in traction power network.

### **IV. CONCLUSION**

In this paper, according to the characteristics of electrified railway harmonic, in accordance with the requirements of active power filter harmonic detection real-time, and accuracy, a harmonic analysis method based on moving-window iteration discrete Fourier transform is proposed, speeds up data update speed, reduces the computation. The simulation results show that when the electric locomotive stable operation, sliding window iteration DFT harmonic detection can accurately detect the traction load current fundamental and harmonic current components to improve the real-time nature of the harmonic current detection. In the case of traction load mutation can also in a power frequency cycle accurately track the changes of load current, it reflects good real-time.

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